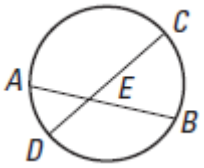
Geometry
Segment Lengths in Circles

Name _____
Date _____ Period _____

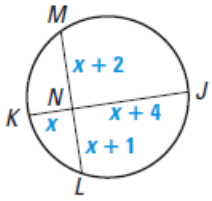
When two chords intersect in the interior of a circle, each chord is divided into two segments that are called segments of the chord.

Segments of Chords Theorem
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



Set up an equation:
$AE(EB) = CE(DE)$

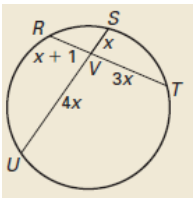
Ex1) Find ML and JK.



$(x+2)(x+1) = x(x+4)$
$x^2+3x+2 = x^2+4x$
$2 = x$

ML: $2+2+2+1 = 7$
JK: $2+2+4 = 8$

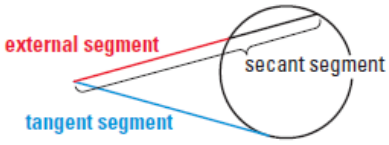
Ex2) Find RT and SU.



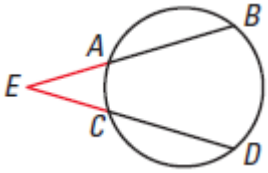
$x(4x) = (x+1)(3x)$
$4x^2 = 3x^2+3x$
$x^2-3x = 0$
$x(x-3) = 0$
$x = 3$

RT: $3+1+3(3) = 13$
SU: $3+4(3) = 15$

A secant segment is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an external segment.

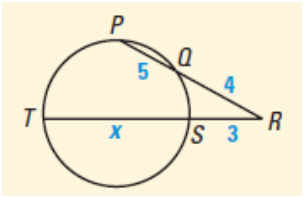


Segments of Secants Theorem
If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



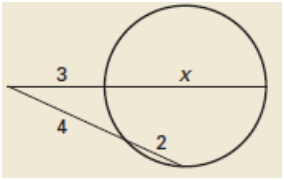
Set up an equation:
$EA(EB) = EC(ED)$

Ex3) What is the value of x?



$4(9) = 3(3+x)$
$36 = 9+3x$
$27 = 3x$
$9 = x$

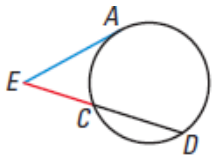
Ex4) What is the value of x?



$3(3+x) = 4(6)$
$9+3x = 24$
$3x = 15$
$x = 5$

Segments of Secants and Tangents Theorem

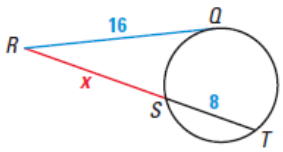
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



Set up an equation:

$EA^2 = EC(ED)$

Ex5) Find RS.



$16^2 = x(x+8)$

$256 = x^2 + 8x$

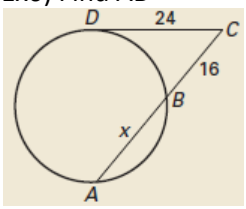
$0 = x^2 + 8x - 256$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{8^2-4(1)(-256)}}{2(1)} =$

$x = \frac{-8 \pm \sqrt{1088}}{2} = \frac{-8 \pm \sqrt{64 \cdot 17}}{2} = \frac{-8 \pm 8\sqrt{17}}{2} = -4 \pm 4\sqrt{17} \approx 12.49$

Ex6) Find AB



$24^2 = 16(16+x)$

$576 = 256 + 16x$

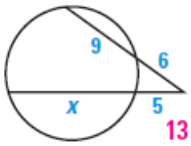
$320 = 16x$

$20 = x$

PRACTICE PROBLEMS

Solve for x.

1)



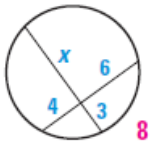
$6(15) = 5(5+x)$

$90 = 25 + 5x$

$65 = 5x$

$13 = x$

2)

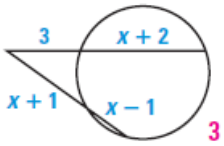


$x(3) = 4(6)$

$3x = 24$

$x = 8$

3)



$3(x+5) = (x+1)(2x)$

$3x + 15 = 2x^2 + 2x$

$0 = 2x^2 - x - 15$

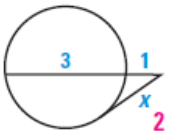
$0 = 2x^2 - 6x + 5x - 15$

$2x(x-3) + 5(x-3)$

$(2x+5)(x-3)$

$x = -\frac{5}{2}$ $x = 3$

4)



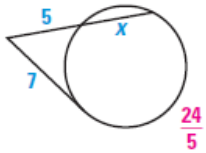
$1(4) = x^2$

$4 = x^2$

$\pm 2 = x$

2

5)



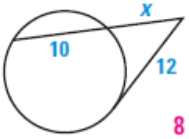
$5(x+5) = 7^2$

$5x + 25 = 49$

$5x = 24$

$x = \frac{24}{5}$

6)



$x(x+10) = 12^2$

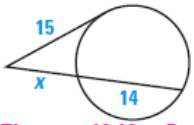
$x^2 + 10x = 144$

$x^2 + 10x - 144 = 0$

$(x+18)(x-8) = 0$

$x = -18$ $x = 8$

7)



Theorem 10.16; $-7 + \sqrt{274}$

$x(x+14) = 15^2$

$x^2 + 14x = 225$

$x^2 + 14x - 225 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-14 \pm \sqrt{(14)^2-4(1)(-225)}}{2}$

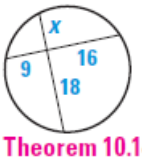
$= \frac{-14 \pm \sqrt{1096}}{2}$

$= \frac{-14 \pm \sqrt{4 \cdot 274}}{2}$

$= \frac{-14 \pm 2\sqrt{274}}{2} = -7 \pm \sqrt{274}$

$= -7 + \sqrt{274}$

8)



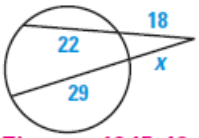
Theorem 10.14; 8

$x(18) = 9(16)$

$18x = 144$

$x = 8$

9)



Theorem 10.15; 16

$18(18+22) = x(29+x)$

$18(40) = 29x + x^2$

$720 = 29x + x^2$

$x^2 + 29x - 720 = 0$

$(x-16)(x+45) = 0$

$x = 16$ $x = -45$