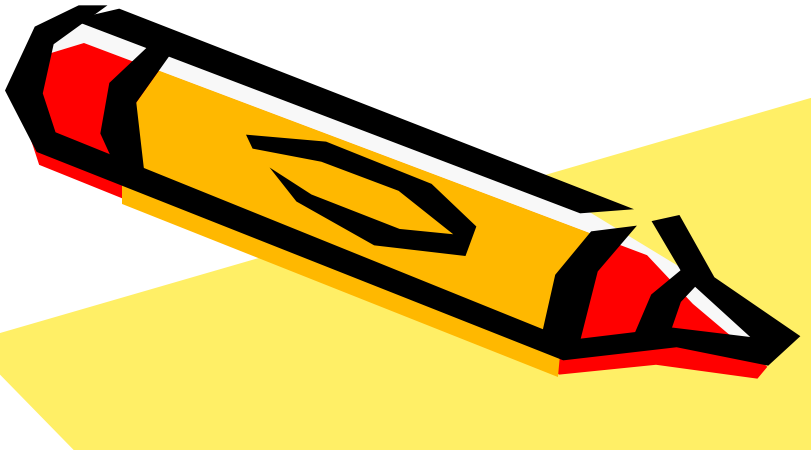
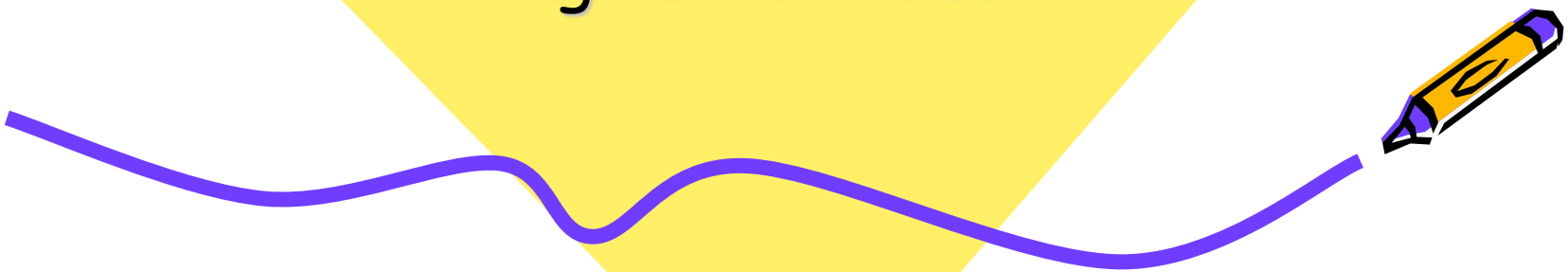


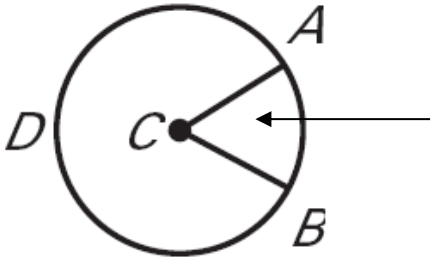
Geometry

Angles and Arcs



Vocabulary

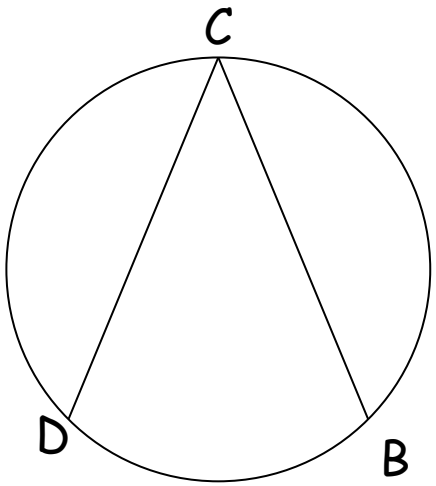
Central Angle - An angle whose vertex is at the center of a circle.



$\angle ACB$: Central Angle

Inscribed Angle – and angle whose vertex is on the circle and the sides are made up of two chords.

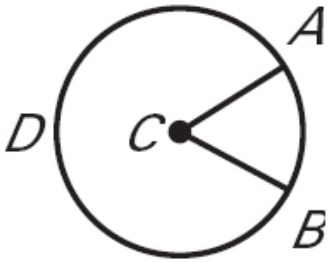
$\angle BCD$ is an inscribed angle



Classifying Arcs

Since a circle has 360 degrees, the arcs of a circle must total 360 degrees as well.

$m\overset{\frown}{AB} + m\overset{\frown}{ADB} = 360$



Types of arcs:

- Minor: less than 180 degrees
- Semicircle: equals 180 degrees
- Major: more than 180 degrees

Naming arcs:

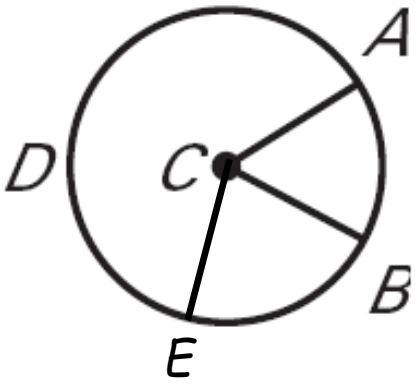
- Minor: two points; the endpoints of the arc
- Semicircle and Major: three points. Two endpoints and one along the way. The middle point indicates which direction to go.

Arc Addition

If 2 arcs are adjacent, we can add them.

Arc Addition works just like segment addition and angle addition.

$m\overset{\frown}{AB} + m\overset{\frown}{BE} = m\overset{\frown}{AE}$



If $m\overset{\frown}{AB} = 50^0$ and $m\overset{\frown}{BE} = 70^0$, then:

$m\overset{\frown}{AE} = 120^0$

$m\overset{\frown}{ADE} = 240^0$

$m\overset{\frown}{ADB} = 310^0$

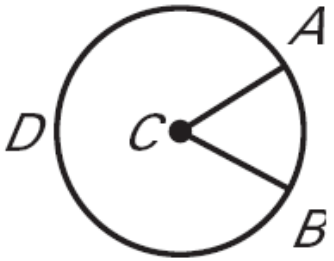
Angle and Arc Relationships

Radii, Chords, Secants, and Tangents can intersect in four ways to forming angles.

The angles can be located at the center of the circle, on the circle, inside the circle, or outside the circle.

Always focus on where the vertex of the circle lies.

Center



angle = its arc

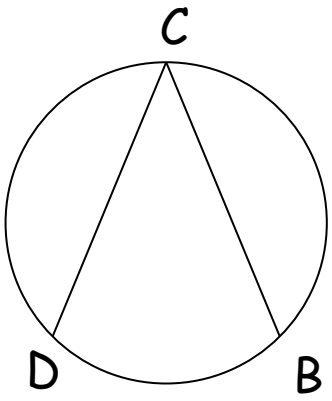
If $m\angle ACB = 40^0$

Then $m\overset{\frown}{AB} = 40^0$

Angle and Arc Relationships

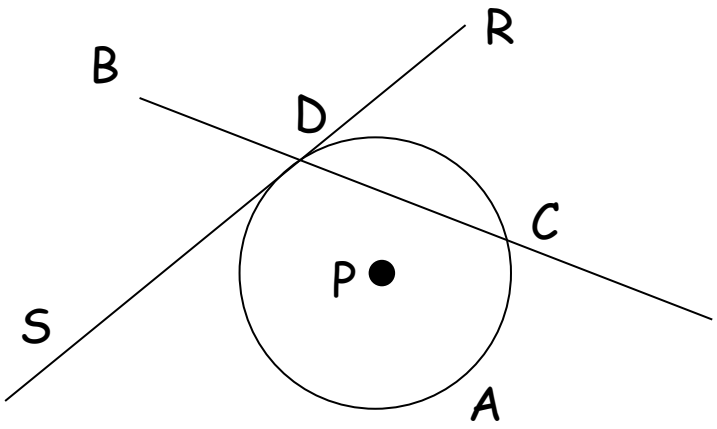
<u>on the circle</u>

angle = $\frac{1}{2}$ (its arc)



$m\angle BCD = \frac{1}{2}\ \overset{\frown}{BD}$

If $m\angle BCD = 40^0$, then $m\overset{\frown}{BD} = 80^0$



$m\angle RDC = \frac{1}{2}\ \overset{\frown}{DC}$

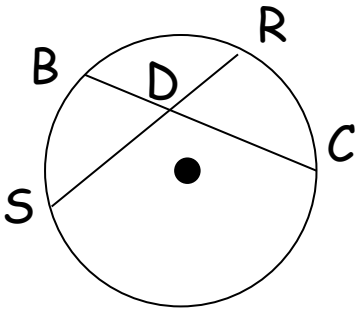
$m\angle SDC = \frac{1}{2}\ \overset{\frown}{DAC}$

$m\angle RDC + m\angle SDC = 180^0$

$m\overset{\frown}{DC} + m\overset{\frown}{DAC} = 360$

Angle and Arc Relationships

<u>inside the circle</u>

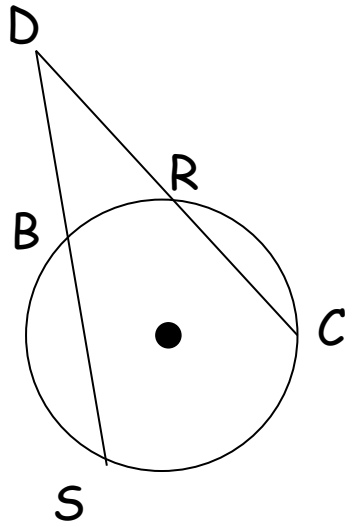


angle = $\frac{1}{2}$ (its arc + its vertical angles arc)

$m\angle RDC = \frac{1}{2}\left(\overset{\frown}{RC} + \overset{\frown}{BS}\right)$

Angle and Arc Relationships

<u>outside the circle</u>



angle = $\frac{1}{2}$ (its big arc - its little arc)

$m\angle RDB = \frac{1}{2}\left(\overset{\frown}{CS} - \overset{\frown}{BR}\right)$

Practice

Find the measure of each indicated arc and classify it as minor, major, or semicircle.

12. $m\overset{\frown}{MN}$

13. $m\overset{\frown}{NQ}$

14. $m\overset{\frown}{NQR}$

15. $m\overset{\frown}{MRP}$

16. $m\overset{\frown}{QR}$

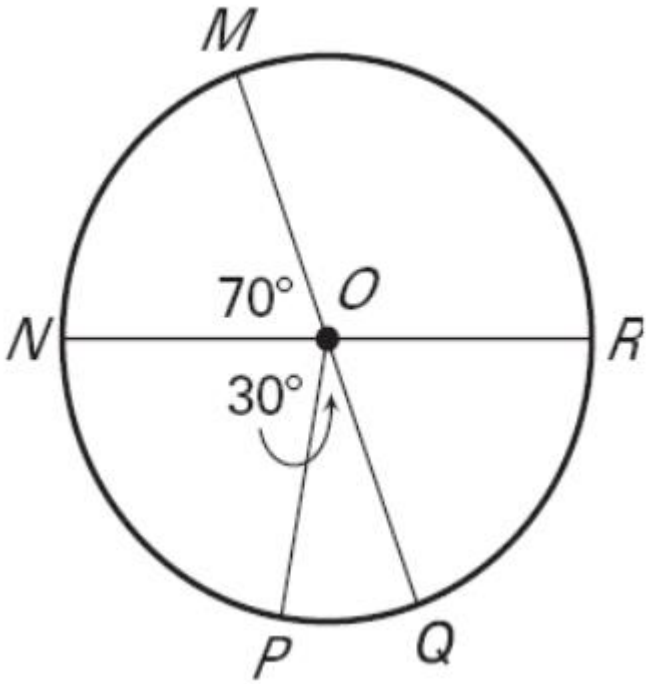
17. $m\overset{\frown}{MR}$

18. $m\overset{\frown}{QMR}$

19. $m\overset{\frown}{PQ}$

20. $m\overset{\frown}{PRN}$

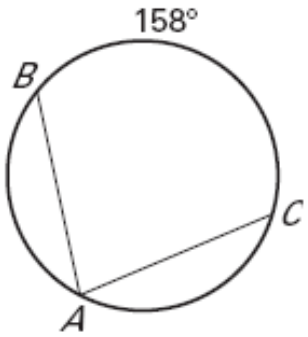
21. $m\overset{\frown}{MQN}$



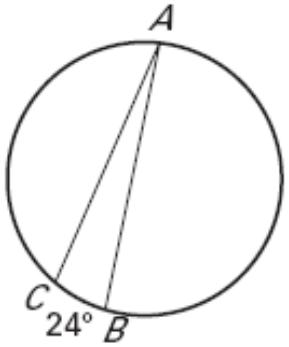
Practice

Find the indicated measure.

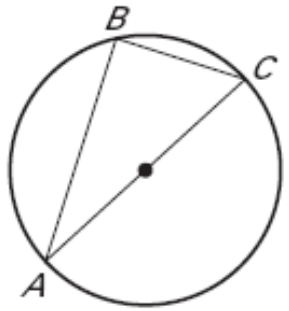
1. $m\angle A$



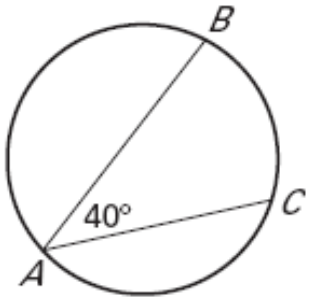
2. $m\angle A$



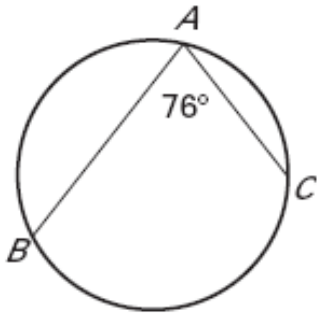
3. $m\angle B$



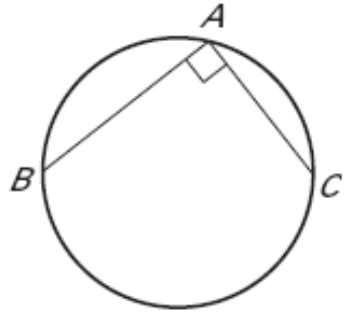
4. $m\overset{\frown}{BC}$



5. $m\overset{\frown}{BC}$



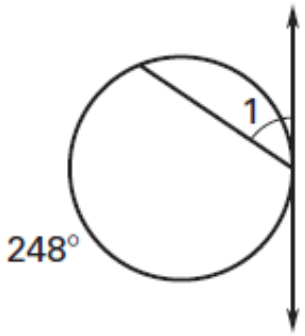
6. $m\overset{\frown}{BC}$



Practice

Find $m\angle 1$.

4.



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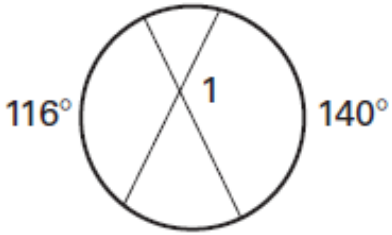
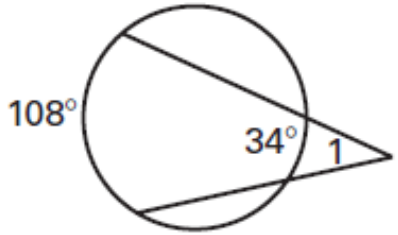


6.



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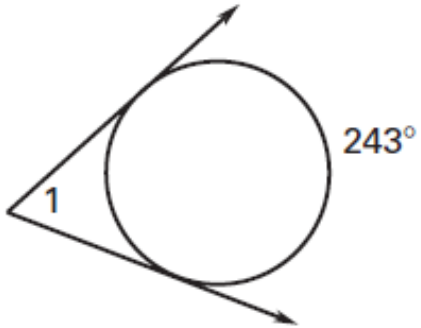

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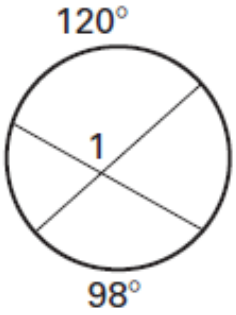

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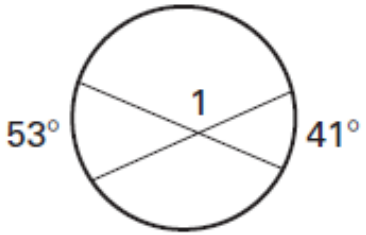
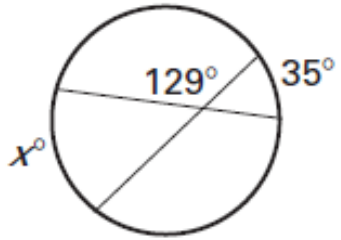


Practice

In Exercises 13–18, find the value of x.

13.



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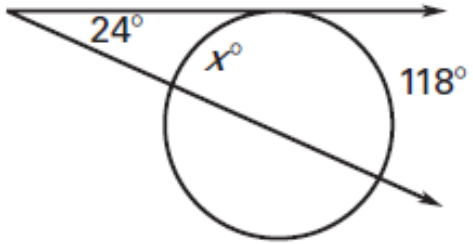

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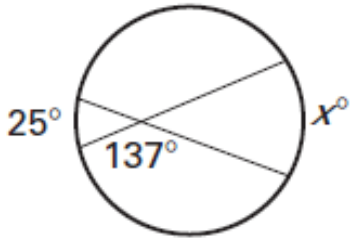

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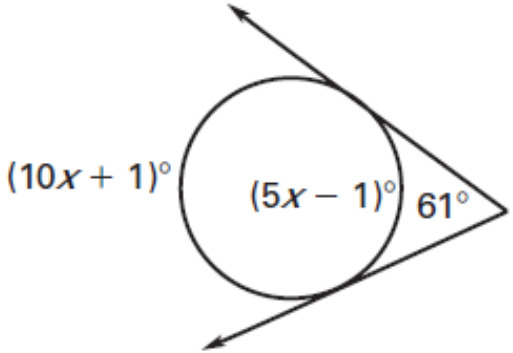
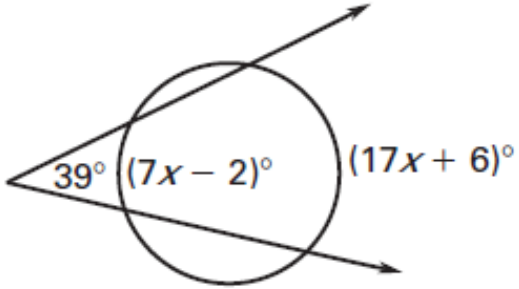
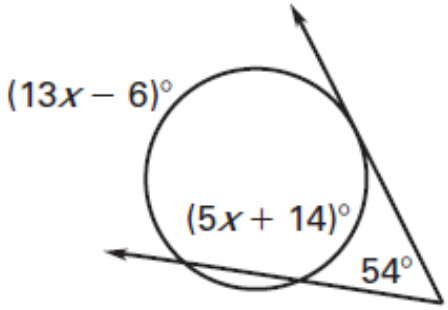


17.



18.



Practice

Find the indicated measure in $\odot M$.

10. $m\angle PNO$

11. $m\angle QNP$

12. $m\overset{\frown}{PQ}$

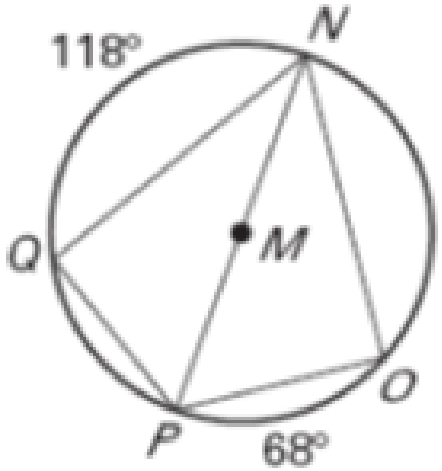
13. $m\overset{\frown}{QO}$

14. $m\angle NMO$

15. $m\overset{\frown}{NOP}$

16. $m\angle QMP$

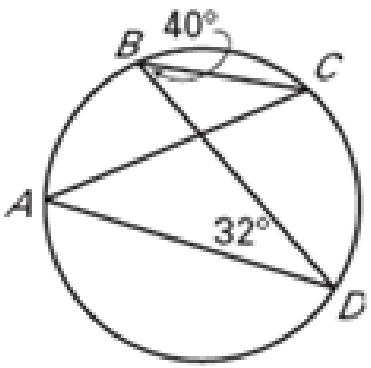
17. $m\overset{\frown}{OQN}$



Practice

Find $m\angle A$ and $m\angle C$.

29.



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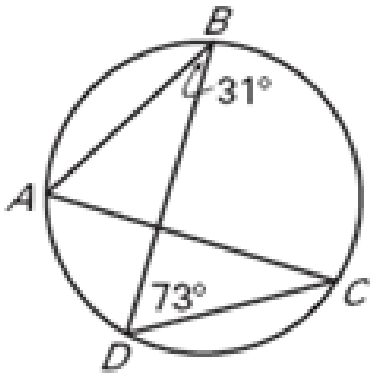
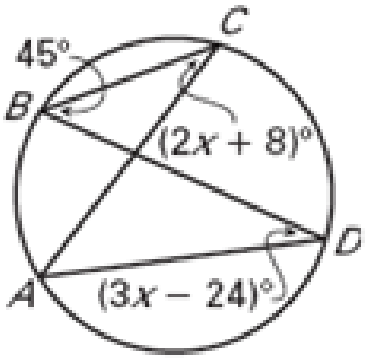
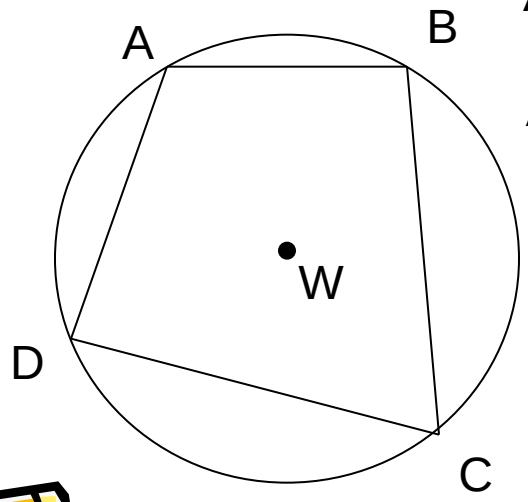


31.



Inscribed Quadrilateral

If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

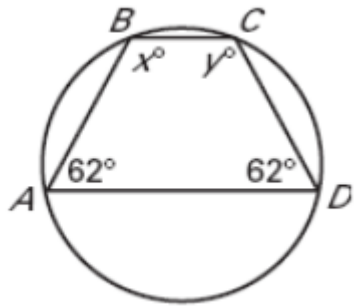


Angles B and D are supplementary
And
Angles A and C are supplementary

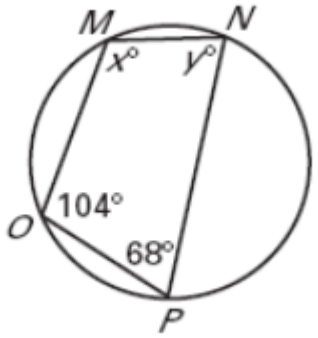
Practice

Find the values of the variables.

23.



24.



25.