Geometry

Segment and Angle Proofs

Vocabulary

Proof:

A logical, step-by-step, explanation that shows the truth of a hypothesis guarantees the truth of the conclusion.

In proofs, our goal is to explain every step of the process, and show that each step is correct by supporting it with mathematical rules and definitions.

This can be done through a formal 2-column proof, or an informal paragraph proof or flow proof.

Proofs

In addition to <u>ALL</u> of the definitions, properties, postulates, and theorems from Geometry, we will be incorporating the following algebraic properties into our work.

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \bullet c = b \bullet c$, $c \neq 0$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.
Distributive Property	$a(b + c) = ab + ac$
Substitution Property of Equality	If $a = b$, then a can be substituted for b (or b for a) in any equation or expression.

Proofs

In addition to ALL of the definitions, properties, postulates, and theorems from Geometry, we will be incorporating the following algebraic properties into our work.

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Algebra Proof (2-column)

Given: $2(x + 5) = 30$ (This is our hypothesis)

Prove: $x = 10$ (This is our conclusion)

Statements (what we do/say)	Reasons (how we know our statement is correct)
$2(X + 5) = 30$	Given
$2x + 10 = 30$	Distribution
$2x = 20$	Subtraction property of equality
$x = 10$	Division rule of equality

Algebra Proof (2-column)

Given: $2(x + 5) = 30$ (This is our hypothesis)

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Geometry Proof (2-column)

Given: AB = CD

Prove: AC = BD

Statements	Reasons
AB = CD	Given
AB+BC=CD+BC	Addition Property of Equality
AB+BC=AC CD+BC=BD	Segment Addition Postulate
AC=BD	Substitution

Geometry Proof (2-column)

Given: AB = CD

Prove: AC = BD

Statements	Reasons
AB = CD	Given
AB+BC=CD+BC	Addition Property of Equality
AB+BC=AC CD+BC=BD	Segment Addition Postulate
AC=BD	Substitution

Geometry Proof (paragraph)

Given: AB = CD

Prove: AC = BD

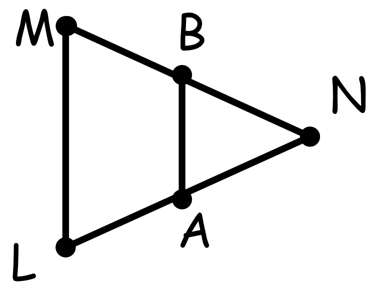
Given AB=CD. AB+BC=CD+BC because if we start with two things that are equal, and we add the same amount (BC) to both, them the results are equal. We know that AB+BC=AC and CD+BC=BD by Segment Addition. By substitution (by substituting the sum for the addend expressions) we can conclude that AC=BD.

Paragraph proofs, and even flow proofs, are much less common than 2-column proofs. They contain the same information, but provide a different way of presenting that information.

Geometry Proof (2-column)

Given: NL = NM

AL = BM



Prove: NA = NB

Statements	Reasons
NL = NM, AL = BM	
NA + AL = NL	
NB + BM = NM	
NA + AL = NB + BM	
NA + BM = NB + BM	
NA = NB	

Geometry Proof

Given: $\angle 1$ and $\angle 2$ are complimentary

$\angle 3$ and $\angle 4$ are complimentary

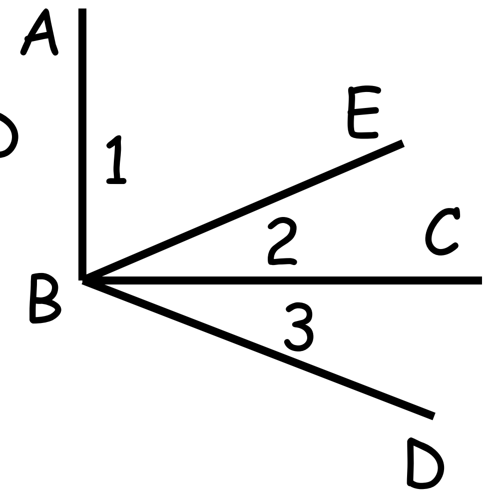
$\angle 2 = \angle 4$

Prove: $\angle 1 = \angle 3$

Geometry Proof

Given: $\overline{AB} \perp \overline{BC}$

$\overline{BC}$ bisects $\angle EBD$



Prove: $\angle 1 + \angle 3 = 90$