Name _____

Partitioning a Directed Line Segment

A **directed line segment** AB is a segment that represents moving from point A to point B. The following example shows how to use slope to find a point on a directed line segment that partitions the segment in a given ratio.

EXAMPLE 1 Partitioning a Directed Line Segment

Find the coordinates of point P along the directed line segment AB so that the ratio of AP to PB is 3 to 2.
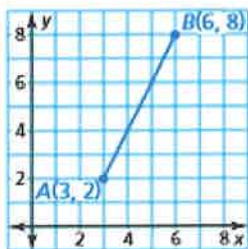
SOLUTION

In order to divide the segment in the ratio 3 to 2, think of dividing, or *partitioning*, the segment into $3 + 2$, or 5 congruent pieces.

Point P is the point that is $\frac{3}{5}$ of the way from point A to point B.

Find the rise and run from point A to point B. Leave the slope in terms of rise and run and do not simplify.

$$\text{slope of } \overline{AB}: \ m = \frac{8-2}{6-3} = \frac{6}{3} = \frac{\text{rise}}{\text{run}}$$

To find the coordinates of point P, add $\frac{3}{5}$ of the run to the x-coordinate of A, and add $\frac{3}{5}$ of the rise to the y-coordinate of A.
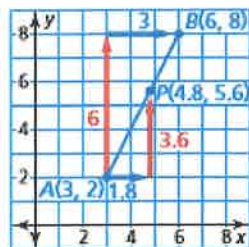
run: $\frac{3}{5}$ of $3 = \frac{3}{5} \cdot 3 = 1.8$

rise: $\frac{3}{5}$ of $6 = \frac{3}{5} \cdot 6 = 3.6$

▶ So, the coordinates of P are

$$(3 + 1.8, 2 + 3.6) = (4.8, 5.6).$$

The ratio of AP to PB is 3 to 2.

In Exercises 3–6, find the coordinates of point P along the directed line segment AB so that AP to PB is the given ratio. (*See Example 1.*)

3. $A(8, 0)$, $B(3, -2)$; 1 to 4   *5 parts : $\frac{1}{5}$ of the way*

4. $A(-2, -4)$, $B(6, 1)$; 3 to 2   *5 parts : $\frac{3}{5}$ of the way*

5. $A(1, 6)$, $B(-2, -3)$; 5 to 1

6. $A(-3, 2)$, $B(5, -4)$; 2 to 6

Handwritten work (right margin):

③ 5 parts   change

$\dfrac{\text{rise}}{\text{run}} \quad \dfrac{-2 \cdot \frac{1}{5}}{-5 \cdot \frac{1}{5}} = \quad \dfrac{-.4}{-1}$

$P(8-1, \ 0 - .4)$

$P(7, -.4)$

④ 5 parts

rise 5 $\frac{3}{5}$ = 3

run 8 $\frac{3}{5}$ = 4.8

$P(-2 + 4.8, \ -4 + 3)$

$P(2.8, -1)$

⑤ rise -9 $\frac{5}{6}$ → -7.5

run -3 $\frac{5}{6}$ → -2.5

$P(-1.5, -1.5)$

⑥ rise -6 · $\frac{2}{8}$ → -1.5

run 8 · $\frac{2}{8}$ → 2

$P(-1, .5)$